STRENGTH PREDICTION FOR GLASS LITES OF ANY SHAPE EXPOSED TO UNIFORM OR NON-UNIFORM LATERAL LOADS

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ABSTRACT

Two standards widely used in North America for strength design of architectural glass rely on a failure prediction model (FPM) that has been fitted to data from tests to failure of full-sized rectangular plates under uniform load. For non-rectangular units or non-uniform loads, however, rational analysis and limiting tensile stress or engineering analysis and judgment are required instead. FPM requires load-induced stresses over the whole glass surface to calculate the probability of brittle fracture. The finite difference program first used to calculate stresses was limited to uniform loads on rectangular plates, the most common of architectural applications. Finite element analysis (FEA) removes these limitations, and with FPM, can usefully contribute to a more rational approach for all glass strength design.

1. INTRODUCTION

Rational analysis of the strength of thin glass plates took a leap forward twenty years ago (1989) with the simultaneous appearance of two North American design standards based on the failure prediction model (FPM) developed over the preceding decade [1-4]. FPM predicts the probability of sudden brittle failure at some pre-existing microscopic surface flaw while under lateral pressure (e.g. wind load). FPM parameters can be adjusted to fit experimental data, and as used in the standards were fitted conservatively to data available at the time.

Surface flaws are assumed to be randomly distributed, and to act as stress multipliers of varying magnitude. Whenever under tensile stress (in the presence of moisture and service temperatures), glass also weakens progressively at surface flaws by replacement of covalent Si bonds with hydrogen bonds between hydroxyl groups:

—Si—Si— +H2O ---> —Si—OH HO—Si—.

The great advantage of FPM is that its underlying theory of cumulative damage leading to sudden brittle failure explains many attributes of glass: 1) reduced strength with increasing plate area; 2) reduced strength with increasing duration and magnitude of load; 3) reduced strength with increasing age of surfaces exposed to degradation; 4) increased strength of glass with residual compressive surface stress (RCSS) locked in by heat treatment or chemical exchanges.

FPM strength predictions require surface stresses as determined by plate geometry and lateral load, load duration, and assumptions about the condition of the glass (new or weathered). The computer program on which the 1989 CGSB and ASTM standards were based was developed specifically for calculating surface stresses, using a finite difference approach, which limited its scope to uniform loads on rectangular plates [5].

Finite element analysis (FEA), once prohibitively expensive and time consuming, is now feasible, though still slower than the venerable finite difference program from the 1980s. FEA is well-suited to handle non-rectangular plates as well as non-uniform loads. Overend *et al* are using a finite element program in their development of a generalized crack growth model offered as an improvement on FPM [6]. Other programs are available that use finite element methods specially adapted to structural analysis of glass structures of virtually any shape and composition, under a wide range of loads and support conditions, but their solutions appear to be based on permissible stresses, and not on FPM [7].

The authors obtained deformation and stress distributions consistent with those of the finite difference program [5] using a general-purpose finite element program [8,9] to solve the same non-linear Von Karman equation for thin plates. Although there is as yet no experimental data to validate the results, this same program and procedure were then used to extend FPM to non-rectangular shapes and non-uniform loads.

Currently, designers of non-rectangular lights can apply some of the insights of FPM by examining rectangular shapes with the same ratio of length to width. The 2004 version of a program to calculate glass strength in accordance with the ASTM E 1300 standard [2] has an "advanced" option that replaces non-rectangular shapes by their enclosing rectangle to guarantee conservative results [10]. Comparisons to design predictions obtained by the authors for a few non-rectangular lights are examined in Section 3.

2. FPM AND ITS RISK FUNCTION

2.1 Loads - stresses - probability of failure

Another remarkable feature of FPM is its economical expression of the key relations between loads, stresses, and probability of failure. The secret of this achievement is to use non-dimensional forms of loads and stresses derived from finite difference program runs to define a "risk function" for deriving the probability of failure.

A program run requires the following inputs: glass properties (modulus of elasticity=E, Poisson's ratio), dimensions of the light (thickness=h, length=a, width=b), and probability distribution parameters (shape=m and scale=S_0), and uniform load=Q. As applied in the two standards [1-2], the parameters to be specified by the designer are the load and the light dimensions — glass properties and probability parameters are fixed.

In non-dimensional form, all these inputs are reduced to two, load (non-dQ) and stress (non-dS).

non-dQ = Q*(ab)² / (Eh⁴) (1)

non-dS = S*(ab) / (Eh²) (2)

Equally important is the non-dimensional expression for the risk function=R, which is determined by three quantities: m, non-dQ, and a/b. Each run is valid for any rectangular light sharing the same non-dQ and a/b!

Probability of failure can therefore be extracted from a table of the risk function R, with M columns of non-DQ and N rows of aspect ratio a/b. The table requires M x N computer runs. Results for intermediate values of non-DQ and a/b can be interpolated.

Additional tables are just as easily prepared for other quantities of interest in design, such as central (maximum) displacement; average displacement; and global maximum principal tensile stress (MPTS) Average displacement is used to compute load sharing in IGU units, and MPTS sheds light on the relation between FPM and the quite different technique of permissible stress design.

2.2 **More about the risk function**

Strength predictions, however arrived at, must deal with random variation in glass strength. Tests to failure of apparently identical plates of annealed glass usually have a coefficient of variation of 20 – 25 percent. What are not apparent are the microscopic surface flaws, assumed to be distributed randomly both spatially and in orientation.

FPM deals with this variability by fitting a two-parameter Weibull distribution, describing the observed probability of failure of test pieces at various pressures. The two parameters set the shape and scale that best fit experimental results. They have been found to vary according to the classification of glass tested: new vs. in-service glass; level of residual compressive surface stress (RCSS).

The risk function is the integration of risk density over that portion of the surface containing micro-flaws under tensile stress normal to their directions. This component of stress is calculated from maximum and minimum principle tensile stress over the whole surface. Thus, each variation affecting stress distribution, for instance a triangular plate versus a rectangular one, requires a whole new tabulation for FPM to work. The same is true for each value or distribution of RCSS [11].

3. SHAPE SHIFTS OR STRESS DESIGN

3.1 Rectangular substitutes

Non-rectangular glazing assembled with larger rectangular units will typically use glass of the same thickness for optical reasons. Shifting to a rectangular shape with the same length to width ratio is another way to use FPM for strength design.

3.2 Equal Area or Enclosing Rectangles

Larger rectangular shapes used to predict the strength of a related non-rectangular shape, produce more conservative designs. Enclosing rectangles are conservative for any shape, but equal area rectangles are not. Table 1 compares design loads for both to those determined by the authors' extended FPM models for non-rectangular shapes.

Table 1: Rectangle loads as percentages of the design load (8/1000) for five different shapes.

Thickness _ Area **mm _ m²**	Load **kPa**	Equal **%**	Enclosing **%**
circle 4_1	2.47	82	68
4_2	1.71	68	55
6_2	2.12	86	72
semi-circle 4_1	1.72	93	78
4_2	1.05	95	81
6_2	1.56	94	77
oval 4_1	1.74	92	77
4_2	1.21	82	70
6_2	1.55	95	78
triangle 4_1	1.79	112	64
4_2	0.97	120	59
6_2	1.65	108	64
trapezoid 4_1	1.78	99	85
4_2	1.08	102	86
6_2	1.60	99	84

Equilateral triangles are weaker than equal area rectangles. Enclosing area rectangles have safety margins ranging from 14 to 45 percent for all the examples in Table 1. FPM tables for non-rectangular shapes would bring significant improvements in consistency and economy to strength design.

3.3 **Comparison of stress and risk functions**

Why not control the risk of failure by keeping the maximum stress on the surface of the plate below a permissible value? This might work if the surface stress was the same everywhere, in every direction, but it isn't. The risk of failure increases with tensile stress normal to the direction of each micro-flaw, so the stress at each point must be reduced as a function of the ratio of minimum to maximum principle stress at each and every point on the surface. Then these modified stresses are integrated over the whole surface to form the risk function.

The next two figures demonstrate the effect of weighting the stresses to form a risk density distribution, which is then integrated to give the risk function. Figure 1 compares stresses to risk densities for the equilateral triangle from Table 1, while Figure 2 does the same for its equal area rectangle.

By FPM design, the equal area rectangle supports 1.2 times the load permitted for the triangle (see Table 1). A maximum stress design, on the other hand, would give a lower load for the rectangle since its maximum stress (26 MPa) exceeds that of the triangle (23 MPa).

The dark areas of the risk density distributions are the most likely regions for failures to start, a fact that can serve as a useful check on the plausibility of FPM, as will be discussed in Section 4.

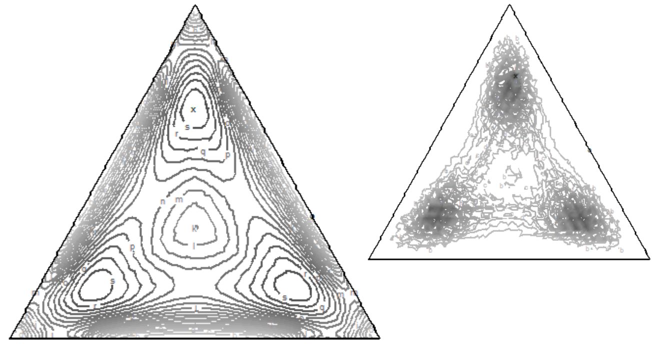


Figure 1: Stresses (greatest in the corner circles, max = 23 MPa) and risk density for a 4mm, 2m² triangle at its design load, 0.97 kPa.

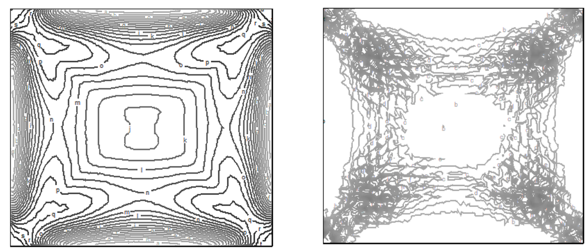


Figure 2: Stresses (greatest at the corners, max = 25.7 MPa) and risk density for a 4mm, 2m² rectangle at its design load of 1.16 kPa.

The next comparison of stress and risk functions was made using a program for manipulating the FPM tables, rather than finite difference or finite element analysis. Figure 3 shows considerable variations of capacity, displacement, and maximum stress with aspect ratio (height / width) for a plate of area 2 m² and thickness 5.66 mm. Unlike the other curves, the risk function is represented in non-dimensional form by its natural logarithm (17.97).

Plots for smaller and larger areas, but keeping the ratio of thickness/(area)½ constant, show similar trends. Variation of capacity with aspect ratio was a contentious issue when FPM was first introduced, but is now accepted, though not easy to demonstrate. As expected, design by global maximum stress will give a different account of this variation with aspect ratio than does FPM.

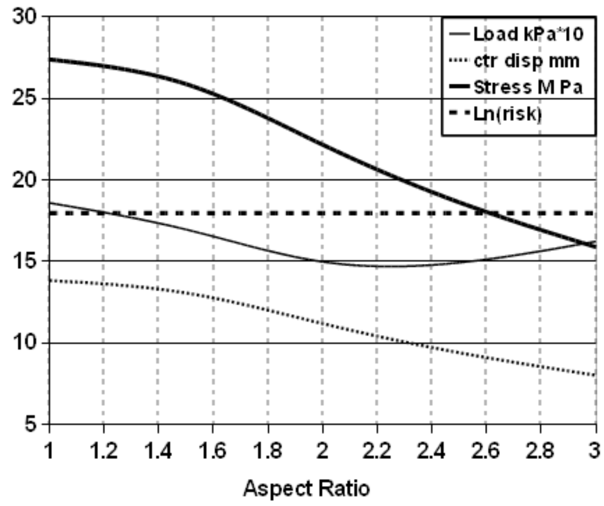


Figure 3: Design (8/1000) capacity, kPa; centre displacement, mm; maximum stress, MPa; and Ln(risk function) for a glass plate of thickness 5.66 mm and area 2 m², plotted against aspect ratio (height/width).

3.4 FPM for point loads

Point and line loads are specified for guard rails in front of glazing approaching floor level in public places. If guard rails are not provided, the glazing itself must be analyzed to demonstrate that those loads can be accommodated. Engineering judgment is used to come up with a critical loaded area (greater than the load patch area in the case of a point load). No such *ad hoc* estimate is required if probability of failure is determined directly from the FPM risk function.

Figure 4 illustrates an FEA/FPM analysis of stress and risk function that confirmed the conclusions of an earlier FEA study using an *ad hoc* critical area estimate of 0.4 m^2.

FPM showed that the load patch area (.01 m^2) contributed 97% of the risk function, which gave a failure probability of 3/1000 for a 1.5 kN load applied for 1 second. The minimum RCSS for heat strengthened glass (24.1 MPa) was used instead of the type factor of 2.0.

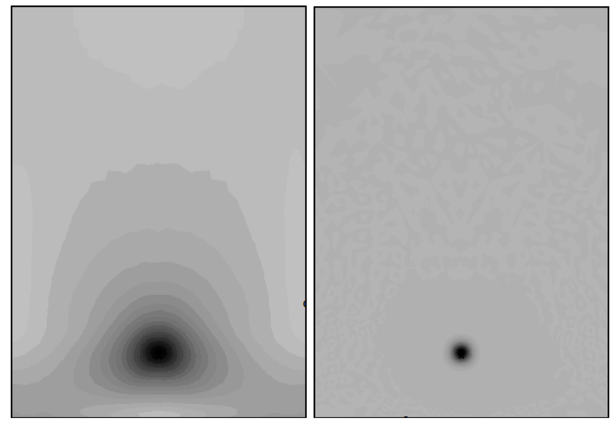


Figure 4: Stress and Risk density distributions for 1.5 KN point load (governing load for the glass guard in this case).

4. RECOMMENDATIONS

4.1 FPM: An accepted procedure

FPM came into favour twenty years ago because it was shown to represent many of the most obvious and essential features of the observed behaviour of glass tested to failure under uniform pressure. Its success is due in large part to tabulations of four key quantities in non-dimensional form that can be applied immediately to a considerable range of practical design problems without the need for expensive and time-consuming numerical analysis in each case.

Now that it is feasible to extend the range of application to non-rectangular glass shapes, and non-uniform load patterns, large-scale experiments seem advisable to confirm that the calculated stresses and deflections are correct, as was done for the finite difference program. Once that is achieved, tabulations for new shapes and loadings can be put into service. In addition, FEA/FPM can then be used with greater confidence for design in one-off situations where shape or load pattern are unusual.

4.2 **Tables for RCSS**

Norville and Morse claim improved accuracy and greater capacity for heat treated glass by replacing the type factors 2, for heat strengthened (HS), and 4, for fully tempered (FT) by new charts appropriate for each type [11]. The larger design loads result from computing the risk function for net tensile stresses after the RCSS is subtracted (minimums allowed are 24.1 MPa for HS and 68.9 MPa for FT).

Norville and Morse have prepared charts for each of ten glass thicknesses (3 – 22 mm) for 9 (HS) plus 7 (FT) RCSS values, a total of 160 charts. The authors, not being particularly fond of working with charts, would prefer tables instead.

When RCSS is used in conjunction with FPM to determine the probability of fracture of tempered or heat-strengthened glass, it is assumed to be uniform, and the lowest values allowed for each category of strengthening are used. In practice, residual stresses from these treatments are neither uniform, nor as low as the allowed minimums.

Non-uniform RCSS is sometimes used deliberately to influence patterns of fracture [12]. FEA offers the possibility of coupling heat flow and mechanical properties to predict non-uniform patterns of RCSS which could then be used with FPM to determine probability of fracture.

4.3 Loads

Loads are even more variable than glass strength. The wind loads specified for glass design are deemed to have a return period of 50 years, i.e. a probability of .02 of being equaled or exceeded in any year. The duration of the design load is currently different for the two standards; 60 seconds in Canada [1] and 3 seconds in the USA [2]. In addition, the Canadian standard uses the limit states design approach, and includes an additional factor of 1.4 to cover possible inaccuracies or variation in the calculation of the wind load.

The 60 second duration is justified in the Canadian standard as the steady state equivalent to a one or two-hour storm's collection of gusts, of which one lasting 1 – 3 seconds is the maximum. The equivalency was calculated using the damage accumulation function inherent in the FPM, on a set of scaled up wind pressure recordings on a high rise building.

Those damage accumulation calculations should be revisited in the light of concerns about the way pressure recordings were substituted for stresses [13]. Interesting observations on the nature of damage accumulation under fluctuating pressures based on wind tunnel data have come to light recently in a Ph.D. dissertation [14], which may lead to an improved approach.

4.4 **Strength of in-service glass**

FPM tables for converting uniform loads to estimates of the probability of failure of rectangular glass plates still use the original compromises for Weibull parameters m and S_0 based on the range of values fitted to the tests to failure of in-service glass available prior to 1989 [15]. The tests of in-service glass revealed significantly higher strengths for surfaces of insulating glass units that faced the sealed air spaces. On the other hand, both exterior and interior surface strengths fell well below that expected for new glass, in some cases by as much as 50 percent.

The consensus reached by the code writers of the CGSB and ASTM standards was a compromise in every sense of the word, and deserves attention here. After considerable discussion, the writers agreed to follow the lead of one prominent manufacturer in settling on in-service, as opposed to new-glass strength, even though only 5 percent of the available tests were made on inner or outer surfaces of glass retrieved after twenty or more years in service (279 out of more than 5000 samples collected world-wide) [15,16].

The compromise reduction in strength currently applies to strengthened glass (RCSS > 0), and in view of its expanding use, there is merit in clarifying whether tensile stress at existing micro-flaws is necessary for strength loss. If RCSS is rarely exceeded in service, the rate of deterioration could be considerably less than the 30 or 40 percent applied to all glass. This might also apply to additional damage created by wind-blown grit.

4.5 **Experimental Validation**

The only sure way to validate the use of any design method, including FPM, is agreement between prediction of failure probability and full-scale experiments. Figure 5 compares FPM prediction to tests of 107 samples of 6mm x 2.36m x 3.78m new float glass. Figure 6 does the same for 47 tests to failure (various sizes) after 15 years in service. In both figures, the ratio of failure load to design load is plotted against probability of failure [16].

Figure 7 shows reasonable agreement between the risk density 'hot spots' and failure origins for 29 4 mm plates 0.924 m x 1.304 m in size.

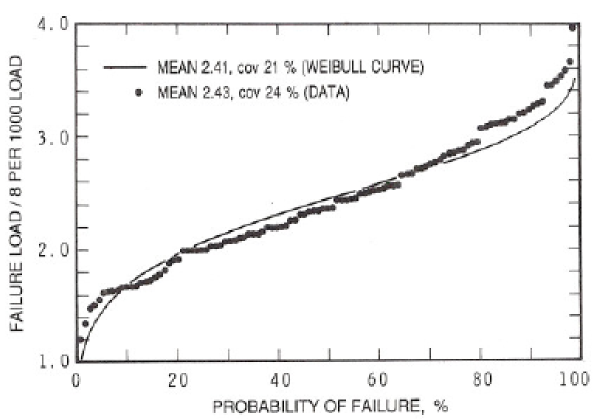


Figure 5. New glass: Ratio of failure load to 8/1000 FPM with m = 7 and S_0 = 40 MPa

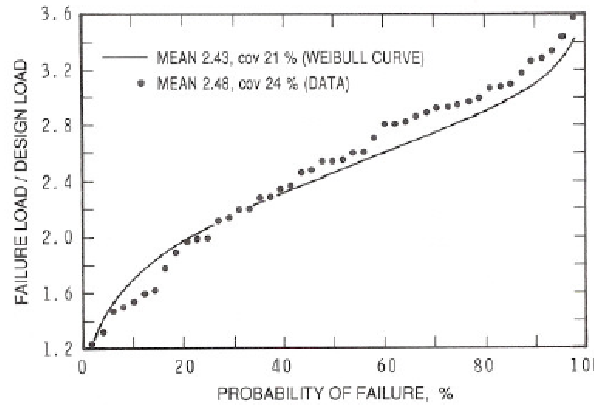
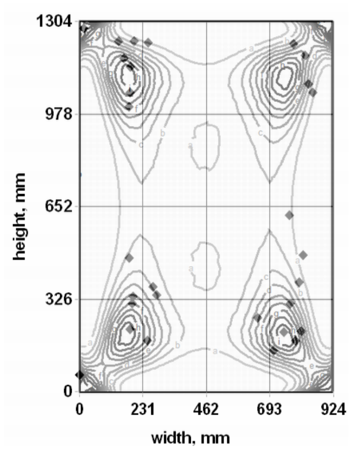


Figure 6. 15-year-old annealed glass: Failure load is divided by the design load from the 1989 CGSB standard, using m = 7 and S_0 = 32.1 MPa.



The 29 diamonds are mostly in or near the highest risk areas of this risk density plot, which applies to a subset of the tests plotted in Figure 6. The 18 other failure origins were on smaller plates.

Figure 7. FPM risk density plot for annealed glass, 4 mm x 924 mm x 1304 mm. Weibull parameters are m = 7 and S_0 = 32.1 MPa.

5. CONCLUSIONS

5.1 Non-rectangular shapes

Additional FPM tables, one set for each new combination of a shape and RCSS level, will well repay the cost of their one-time preparation, even though many FEA runs are involved.

5.2 Non-uniform loads

Dedicated FEA runs will be required for each configuration. FPM is highly recommended for estimating probability of failure.

5.3 **FPM improvements and validation**

Validation by full-scale and laboratory experiments has been the foundation of FPM from the beginning. Improvements often follow from comparisons with tests, and each extension, such as non-rectangular shapes, should be followed up by tests. Strength degradation urgently requires investigation. Other much needed improvements (not discussed in this paper) include consideration of dynamic impact loading and failures originating at edge flaws [16].

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References

[1] *Structural Design of Glass for Buildings*, (1989) CAN/CGSB-12.20-M89, Canada. Clause 6.4.1, page 6.

[2] *Standard Practice for Determining Load Resistance of Glass in Buildings*, (2002) ASTM E 1300 – 02, ASTM International, PA, USA. Clause 5.4, page 3.

[3] Brown, W.G., (1974) "A Practicable Formulation for the Strength of Glass and Its Special Application to Large Plates," *Publication No. NRC 14372*, National Research Council of Canada, Ottawa, ON Canada.

[4] Beason, W.L., Morgan, J.R. (1984) "Glass Failure Prediction Model," *Journal of Structural Engineering,* 110(2) pp. 197-212.

[5] Vallabhan, C.V.G., Wang, B.Y-T., Non-linear Analysis of Rectangular Glass Plates by Finite Difference Method," Institute for Disaster Research, Texas Tech University, Lubbock TX, June 1981.

[6] Overend, M., Parke, G.A.R. and Buhagiar, D. (2007) "Predicting Failure in Glass – a General Crack Growth Model", *Journal of Structural Engineering* 133(8) pp. 1146 -1155.

[7] *SJ MEPLA – Software for Structural Glass Design,* (2008) SJ Software GmbH, Haarfhotstr. 52, D-52080 Aachen Germany www.mepla.net

[8] *FlexPDE 6 – finite element model builder for Partial Differential Equations,* (2009) PDE Solutions Inc. WA USA www.pdesolutions.com

[9] Brown, J., (2006) System of 4th order differential equations, FlexPDE Forum

[10] *Standards Design Group Inc – Window Glass Design*, (2004) Lubbock, TX, USA www.standardsdesign.com

[11] Norville, H.S., Morse, S.J. (2009) A Model to Describe the Load Resistance (Strength) of Heat-Treated Architectural Glass, *Glass Performance Days 2009* Tampere, Finland 5 pages www.gpd.fi

[12] *Producing glass sheets having improved break patterns* (1980) Thomas *et al* US Patent 4,236,909

[13] Dalgliesh, W.A., (1980), "Assessment of Wind Loads for Glazing Design," Proceedings, IAHR/IUTAM Symposium on Practical Experiences with Flow-Induced Vibrations. Naudashcher, E. and Rockwell, D. (Eds.), Springer-Verlag, Berlin Heidelberg, p. 606-708.

[14] Gavanski, E., (2009), "Behaviour of Glass Plates under Wind Loads," Department of Civil and Environmental Engineering, Faculty of Engineering Science, University of Western Ontario London, ON, Canada.

[15] Committee on Window Glass Research, (1985), "A Basis for the Design of Window Glass under Wind Pressure," Minor, J.E., Dalgliesh, W.A., (Co-chairmen) Glass Research and Testing Laboratory, Texas Tech University, Lubbock, TX, USA

[16] Dalgliesh, W.A., Taylor, D.A. (1990), "The Strength and Testing of Window Glass," Canadian Journal of Civil Engineering, 17(5) p. 752-762 (IRC Paper No. 1673)